Spark's Guide: Quadratic Functions

Your one-stop guide to Quadratic Functions

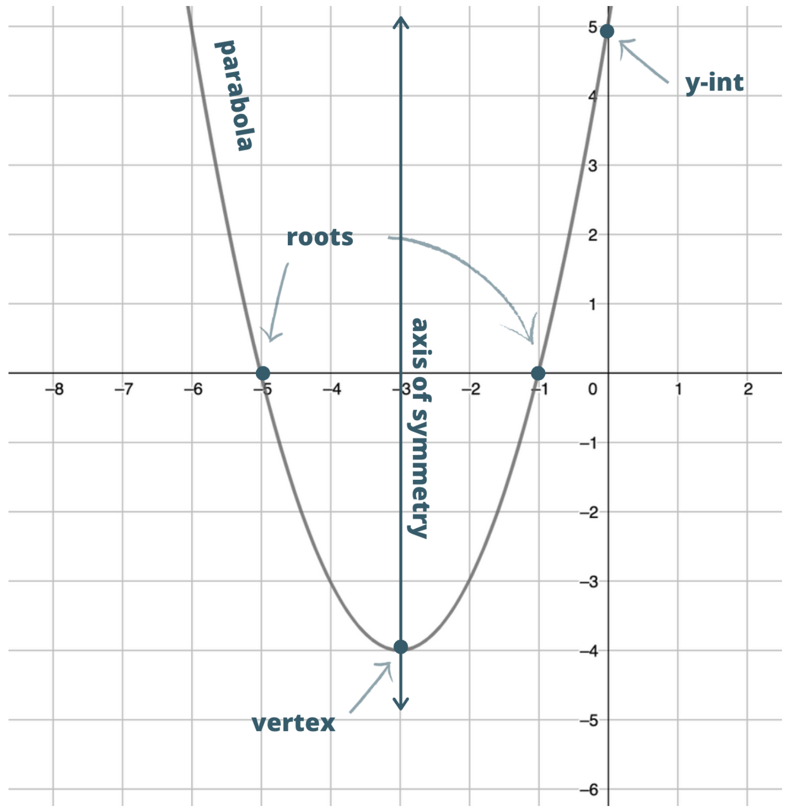
SPARK TUTORS

Let's start with some important vocabulary

Standard form: $y = ax^2 + bx + c$ where a is the multiplier, c is the y-intercept

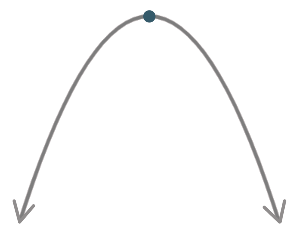
Vertex form: $y = a(x - h)^2 + k$ where $(h, k)$is the vertex.

Factored form: $y = a(x - x_1)(x - x_2)$ where x_1 and x_2 are roots



If *a* is **positive**, the parabola opens up
The vertex is a **local minimum**

If *a* is **negative**, the parabola opens down
The vertex is a **local maximum**



Graphing in standard form:

$$y = ax^2 + bx + c$$

Given $y = ax^2 + bx + c$

1. Calculate the vertex (h, k):

$$h = \frac{-b}{2a}$$

$$k = ah^2 + bh + c$$

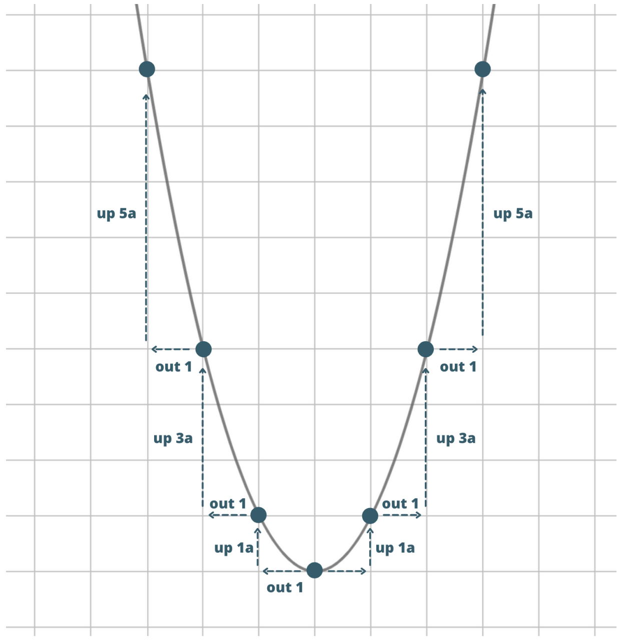
2. Graph the vertex

3. Identify the multiplier, a

4. Graph the pattern, **1a, 3a, 5a**.

   → Out 1, up 1a; out 1, up 3a; out 1, up 5a.

5. Graph the parabola



<u>Example:</u> Graph the following quadratic function.

$$y = -x^2 + 4x - 1$$

1. Calculate the vertex (h, k):

$$h = \frac{-b}{2a} = \frac{-4}{2\cdot(-1)} = 2$$

$$k = ah^2 + bh + c = -(2)^2 + 4(2) - 1 = 3$$

   → so the vertex is (2, 3)

2. Graph the vertex

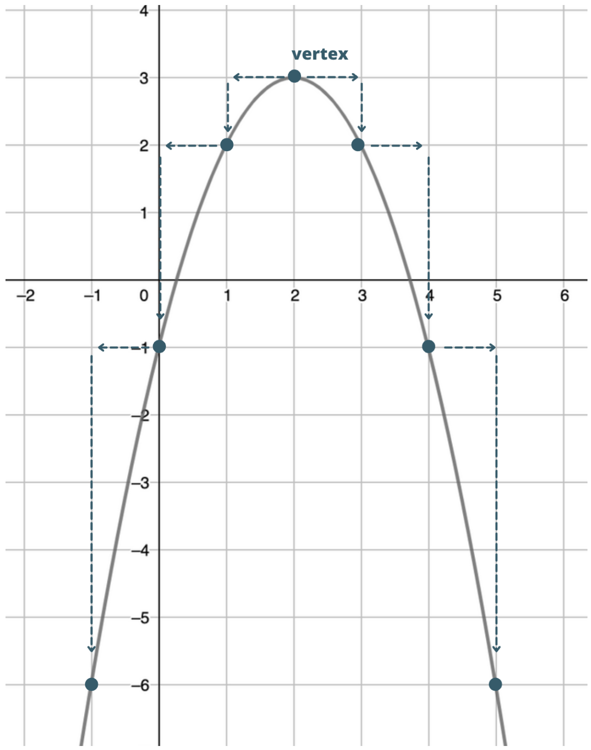
3. Identify the multiplier, $a = -1$

   → Therefore, the pattern is -1, -3, -5

   → Out 1, down 1. Out 1, down 3. Out 1, down 5.

4. Graph the pattern

5. Graph the parabola



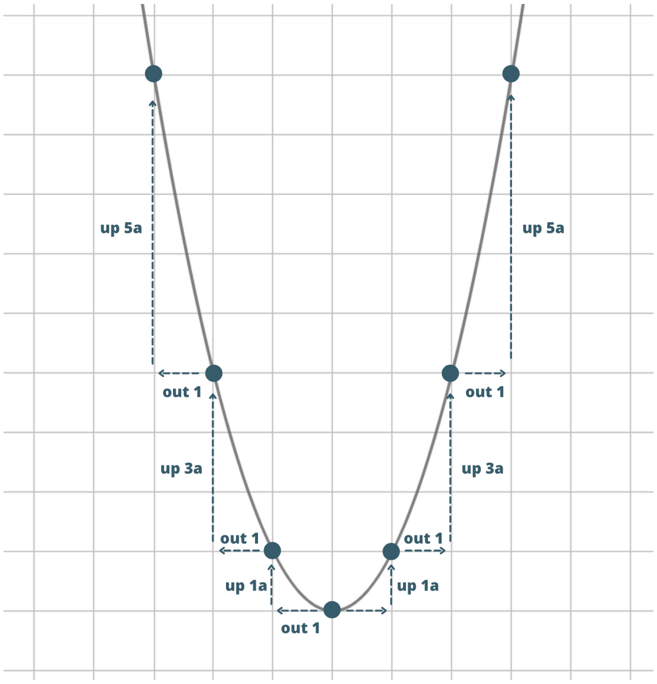
Graphing in vertex form:

$$y = a(x - h)^2 + k$$

Given $y = a(x - h)^2 + k$

1. In the equation, identify the vertex, (*h, k*)
2. Graph the vertex
3. Identify the multiplier, a
4. Graph the pattern, **1a, 3a, 5a**.

   → Out 1, up 1a; out 1, up 3a; out 1, up 5a.
5. Graph the parabola



Example: Graph the following quadratic function.

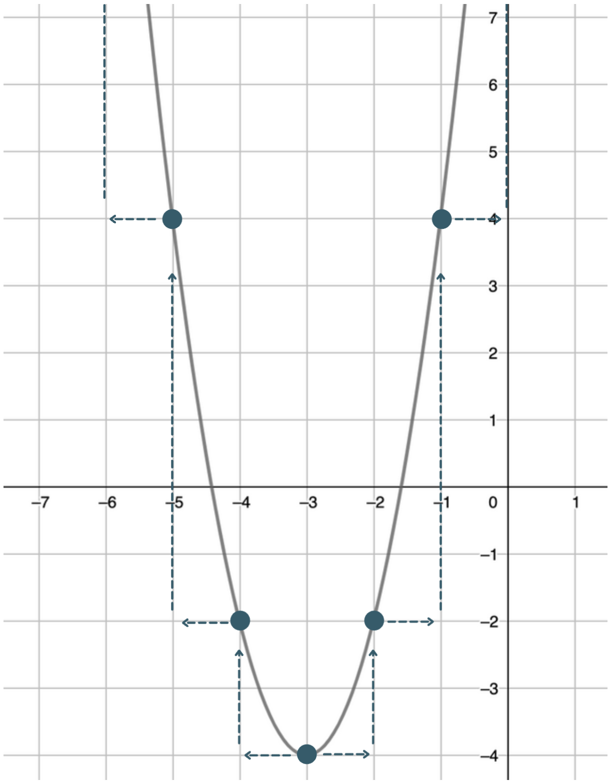
$$y = 2(x + 2)^2 - 3$$

1. In the equation, identify the vertex, (*h, k*)

   → the vertex is $(-2, \ -3)$
2. Graph the vertex
3. Identify the multiplier, *a*

   → *a* = 2, so the pattern is 2, 6, 10.

   → Out 1, up 2; out 1, up 6; out 1, up 10.
4. Graph the pattern
5. Graph the parabola



Completing the square when $a = 1$:

This method is used to convert from **standard form** to **vertex form**.

Steps	Work
1. Start with your equation.	$y = x^2 + 6x - 1$
2. Move c to the right and add gaps.	$y = x^2 + 6x + \_\_\_ - \_\_\_ - 1$
3. Find $p \rightarrow p = \frac{b}{2}$	$p = 3$
4. Fill gaps with p^2	$y = x^2 + 6x + 9 - 9 - 1$
5. The first three terms factor into $(x + p)^2$	$y = (x + 3)^2 - 9 - 1$
6. Add the remaining terms	$y = (x + 3)^2 - 10$

Done!

Completing the square when $a \neq 1$

Buckle up! This one's tricky!

Steps	Work
1. Start with your equation	$y = 2x^2 + 16x + 25$
2. Factor *a* out of the first two terms.	$y = 2(x^2 + 8x) + 25$
3. Add the +gap within the parentheses and the -gap outside the parentheses	$y = 2(x^2 + 8x \ + \_\_\_) - \_\_\_ + 25$
4. Find $p \rightarrow p = \frac{b}{2}$	$p = 4$
5. Fill gaps with p^2 → *be careful here, because the outside gap must be multiplied by a*	$y = 2(x^2 + 8x + 16) - 32 + 25$
6. The parentheses factor into $(x + p)^2$	$y = 2(x + 4)^2 - 32 + 25$
7. Add the remaining terms	$y = 2(x + 4)^2 - 7$

Done!

Factoring:

This method is used to convert from **standard form** to **factored form**.

Given: $y = x^2 + bx + c$

1. Think of two numbers that add to b and multiply to c.
2. Fill the gaps with those numbers.

Example: Put the following function in factored form.

$$y = x^2 - 2x - 35$$

1. Think of two numbers that add to -2 and multiply to -35.

   Here, it is -7 and 5; they add to -2 and multiply to -35.

2. Fill the gaps with those numbers.

   $y = (x - 7)(x + 5)$

Done!